THE TRADEOFF BETWEEN DIVERSITY GAIN AND INTERFERENCE SUPPRESSION VIA BEAMFORMING IN A CDMA SYSTEM

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ABSTRACT

In this paper, the uplink of an asynchronous Direct Sequence Code Division Multiple Access (DS-CDMA) system with multiple antennas at both the transmitter and the receiver is considered. We analyze the system performance over a spatially correlated Rayleigh fading channel with multiple access interference (MAI). Assuming perfect channel knowledge available at the transmitter, Maximal Ratio Transmission (MRT) is employed to weight the transmitted signal optimally in terms of combating signal fading. Further, adaptive beamforming reception is adopted to suppress MAI and also to combat the fading. We examine the effect of varying the number of transmit and receive antennas on the diversity gain and interference suppression.

INTRODUCTION

By utilizing antenna arrays at both the transmitter as well as the receiver, the limitations of the radio channel may be reduced and the data rates increased. This triggered the development of space-time codes that employ both the spatial and the temporal dimensions to achieve a significant portion of the channel capacity calculated in [1]. Common to the space-time coding schemes [2], [3] is that they do not exploit channel knowledge at the transmitter. Channel information, if it is available, should of course be utilized to maximize the performance. Beamforming is one solution when there is a dominant direction-of-arrival (DOA) for the signal-of-interest. For a transmit array, the channel information is used to focus as much energy in the direction of the receiver as possible. For a receive array, the gain of the antenna is maximized in the direction of the path with the strongest power. Compared with the space-time coding schemes, beamforming is preferred in terms of complexity.

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Furthermore, in a multiple access channel, where K_u users each with L_T transmit antennas try to communicate with a common receiver with L_R receive antennas, beamforming is not only sufficient but also necessary for achieving the so-called sum capacity of multiple access channels if the number of users is much larger than the number of receive antennas [4]. This latter condition generally holds in a CDMA system.

In this paper, the uplink of an asynchronous DS-CDMA system with multiple antennas at both the transmitter and the receiver is considered. Assuming perfect channel knowledge available at the transmitter, Maximal Ratio Transmission (MRT) is employed to weight the transmitted signal optimally in terms of combating signal fading. Adaptive beamforming reception is adopted to suppress MAI and also to combat the fading. We analyze the system performance over a spatially correlated Rayleigh fading channel with multiple access interference (MAI), and evaluate the antenna array performance with joint fading reduction and MAI suppression. The detailed organization of the paper is as follows. The system model and channel model used in the study are described in Section II. Section III presents the analysis of system performance, and is followed by some numerical results and discussions in Section IV. Finally, conclusions are drawn in Section V.

SYSTEM MODEL

We describe a system model exploiting multiple antennas in a single cell CDMA system. Assume that both the mobiles and the base station use an antenna array to transmit and receive signals, where each mobile has an antenna array, of size L_T, used for maximal ratio transmission [5], and the base station has an antenna array, of size L_R, used for adaptive beamforming reception.

For the block diagram shown in Figure 1, the transmitted signal of user k is given by

$$\begin{aligned}\underline{s}_k(t) &= \sqrt{P_k}\sum_{n=-\infty}^{\infty} c_n^{(k)} h(t-nT_c) \\ &\cdot u_{\lfloor n/N_s \rfloor}^{(k)} \underline{v}_k \exp\left[j(\omega t+\theta_k)\right]\end{aligned} \tag{1}$$

where $u_i^{(k)}$ is the ith data symbol of user k, $\underline{v}_k$ is a transmission weight vector for user k, ω is the carrier frequency, θ_k is a random carrier phase associated with user k and is uniformly distributed over $[0, 2\pi)$, the spreading sequences of the interfering users, $c_n^{(k)}$, $k = 2, \cdots, K$, are assumed to be i.i.d. random variables taking values ± 1 with equal probability, while that of the desired user, $c_n^{(1)}$, is taken to be deterministic, $h(t)$ is the impulse response of the baseband chip wave-shaping filter, and $1/T_c$ is the chip rate of a band-limited DS-CDMA system. We also define $x(t) = F^{-1}|H(f)|^2$ and assume that $x(t)$ satisfies the Nyquist criterion, i.e., $x(nT_c) = \delta(n)$. The processing gain is defined as $N_s = T_s/T_c$, and is taken to be much smaller than the period of the spreading sequence, where T_s is the symbol duration.

The channel model is taken to be a slowly varying Rayleigh fading channel, with transfer function $\xi_{r,l}^{(k)} = \alpha_{r,l}^{(k)} \exp(j\beta_{r,l}^{(k)})$, for $r = 1, \cdots, L_R$ and $l = 1, \cdots, L_T$, where l is the index for the transmit antennas and r is the index for the receive antennas. We assume that $\left\{\alpha_{r,l}^{(k)}\right\}$ and $\left\{\beta_{r,l}^{(k)}\right\}$ are statistically independent for different users, and that $\left\{\alpha_{r,l}^{(k)}\right\}$ and $\left\{\beta_{r,l}^{(k)}\right\}$ are, respectively, i.i.d Rayleigh random variables with a unit second moment, and uniform random variables over $[0, 2\pi)$ for different transmit antennas. However, the array gain and the phase of the different elements in the receive antenna array might be correlated, where the correlation is determined by parameters such as direction of arrival $\phi_l^{(k)}$, angular spread $\Delta_l^{(k)}$, spacing between neighboring receive antennas D_r and the wavelength of the carrier signal λ, as shown in Fig. 3.

Specifically, using the model in Fig. 3, a closed-form spatial correlation formula is given by [6],

$$\begin{aligned}E\left\{(\zeta_{i,l}^{(k)})(\zeta_{j,l}^{(k)})^*\right\} &= Rs(\Delta_l^{(k)}, \phi_l^{(k)}, D_r, \lambda) \\ &= \left[Rs_l^I(i,j) + jRs_l^Q(i,j)\right]\end{aligned} \tag{2}$$

where $Rs_l^I(i,j)$ and $Rs_l^Q(i,j)$ are given by

$$\begin{aligned}Rs_l^I(i,j) &= J_0\left(\frac{2\pi D_r|i-j|}{\lambda}\right) + 2\sum_{n=1}^{\infty} J_{2n} \\ &\cdot \left(\frac{2\pi D_r|i-j|}{\lambda}\right)\cos(2n\phi_l^{(k)}) sinc(2n\Delta_l^{(k)})\end{aligned} \tag{3}$$

and

$$\begin{aligned}Rs_l^Q(i,j) &= 2\sum_{n=0}^{\infty} J_{2n+1}\left(\frac{2\pi D_r|i-j|}{\lambda}\right) \\ &\cdot \sin((2n+1)\phi_l^{(k)}) sinc((2n+1)\Delta_l^{(k)})\end{aligned} \tag{4}$$

respectively, for $l = 1, \cdots, L_T$, and where the J_n's are Bessel functions of integer order. When this correlation is high, the signals at the antennas tend to fade at the same time, and the diversity benefit of antenna arrays against fading is significantly reduced. On the other hand, because independent fading is not required for interference suppression, antenna arrays can suppress interference even with complete correlation. Thus, we need to evaluate the antenna array performance with joint fading reduction and interference suppression.

We define a channel matrix H_k by putting the channel gain of each transmit and receive antenna pair into a matrix of size $L_R \times L_T$. That is to say, the i,jth entry in H_k is $\xi_{i,j}^{(k)}$. Thus, the received signal vector in the antenna array is obtained as

$$\underline{r}(t) = \sum_{k=1}^{K} H_k \underline{s}_k(t-\tau_k) + \underline{n}_w(t) \tag{5}$$

where τ_k is an arbitrary time delay uniformly distributed over $[0, T_s]$, and $\underline{n}_w(t)$ is the AWGN vector added to the receive antenna array such that each of its elements is a zero-mean complex Gaussian random process with two-sided spectral density η_0. An asynchronous DS-CDMA system is assumed, but the receiver is synchronized to the desired transmission, say that of user 1; thus, we assume that the power and delay of the desired signal are, respectively, $P_1 = 1$ and $\tau_1 = 0$, without loss of generality.

PERFORMANCE ANALYSIS

We evaluate the performance of the first user. Perfect carrier, code, and bit synchronization are assumed in the receiver side, as shown in Fig 2. The output of the correlator during the ith symbol interval, $\underline{z}_1(i)$, obtained by summing the corresponding despread N_s chip-matched filter output, is given by

$$\begin{aligned}\underline{z}_1(i) &= \frac{1}{N_s}\sum_{n'=(i-1)N_s}^{iN_s-1} c_{n'}^{(1)} \\ &\quad\cdot \left[(r(t)\exp(-jwt))\star h(-t)\right]_{t=n'T_c} \\ &= \frac{1}{N_s}G_1\underline{v}_1\sum_{n'=(i-1)N_s}^{iN_s-1} c_{n'}^{(1)}\sum_{n=-\infty}^{\infty} u_{\lfloor n/N_s\rfloor}^{(1)}c_n^{(1)} \\ &\quad\cdot\int_{-\infty}^{\infty} h((n'-n)T_c+\tau)h(\tau)d\tau + \underline{I}_1(i) + \underline{N}_1(i) \\ &= u_i^{(1)}G_1\underline{v}_1 + \underline{I}_1(i) + \underline{N}_1(i)\end{aligned} \tag{6}$$

where $\star$ represents convolution, $G_k = H_k\exp(j\theta_k)$ for $k = 1, \cdots, K$.

$$\underline{S}_1(i) = u_i^{(1)}G_1\underline{v}_1$$

is the signal component for the desired user,

$$\begin{aligned}\underline{N}_1(i) &= \frac{1}{N_s}\sum_{n=(i-1)N_s}^{iN_s-1} c_n^{(1)} \\ &\quad\cdot\left\{[\underline{n}_w(t)\exp(-jwt)]\star h(-t)\right\}_{|t=nT_c}\end{aligned} \tag{7}$$

is the component due to thermal noise, and

$$\begin{aligned}\underline{I}_1(i) &= \sum_{k=2}^{K}\frac{\sqrt{P_k}}{N_s}\sum_{n'=(i-1)N_s}^{iN_s-1} c_{n'}^{(1)}\sum_{n=-\infty}^{\infty} \\ &\quad\cdot u_{\lfloor n/N_s\rfloor}^{(k)}c_n^{(k)}x((n'-Mn)T_c-\tau_k)G_k\underline{v}_k \\ &= \sum_{k=2}^{K}\frac{\sqrt{P_k}}{N_s}R_{k,1}(i)G_k\underline{v}_k\end{aligned} \tag{8}$$

is the multiple access interference. In (8),

$$\begin{aligned}R_{k,1}(i) &= \sum_{n'=(i-1)N_s}^{iN_s-1} c_{n'}^{(1)} \\ &\quad\cdot\sum_{n=-\infty}^{\infty}\mu_n^{(k)}x((n'-n)T_c-\tau_k)\end{aligned} \tag{9}$$

is the cross-correlation function of the spreading signal between user k and user 1 during the ith symbol interval. Here we define $\mu_n^{(k)} = u_{\lfloor n/N_s\rfloor}^{(k)}c_n^{(k)}$, absorbing $u_{\lfloor n/N_s\rfloor}^{(k)}$ into $c_n^{(k)}$, since both are random variables taking values of ± 1 with equal probability. By the Liapounoff version of the central limit theorem, $\underline{I}_1(i)$ can be modeled as an asymptotically complex Gaussian vector as long as the following condition is satisfied [8]: $\sum_{n=-\infty}^{\infty}|x(nT_c-\tau)| < \infty$ for all τ, where $0 \le \tau < T_c$.

The correlator outputs from each receive antenna are combined with the beamforming vector $\underline{w}_1 = [w_{1,1}, \cdots, w_{1,L_R}]^T$ to produce an estimate of the transmitted symbol of the desired user. The estimated data symbol can be represented as

$$\widehat{u}_{1,i} = \underline{w}_1^H\underline{z}_1(i) = S_{1,i} + I_{1,i} + N_{1,i}. \tag{10}$$

Now we proceed to determine the optimum transmit and receive weight vectors $\underline{v}_1$ and $\underline{w}_1$ respectively, for the desired user. Since the MAI, $\underline{I}_1(i)$, can be modeled as an asymptotically zero-mean complex Gaussian vector, and is independent of the AWGN vector $\underline{N}_1(i)$, the conditional SINR, γ_i, of the estimated data, $\widehat{u}_{1,i}$, conditioned on G_1, is given by

$$\begin{aligned}\gamma_i &= \frac{|S_{1,i}|^2}{Var(N_{1,i}) + Var(I_{1,i})} \\ &= \frac{\underline{w}_1^H G_1\underline{v}_1\underline{v}_1^H G_1^H\underline{w}_1}{\underline{w}_1^H E\left\{\underline{N}_1(i)\underline{N}_1^H(i) + \underline{I}_1(i)\underline{I}_1^H(i)\right\}\underline{w}_1} \\ &= \frac{\left|\underline{w}_1^H G_1\underline{v}_1\right|^2}{\underline{w}_1^H\left[\frac{\eta_0}{N_s}I_{L_R} + \sum_{k=2}^{K}\frac{P_k}{N_s}R_I(0)(V_k)^H R_I^{(k)}V_k\right]\underline{w}_1}\end{aligned} \tag{11}$$

Refer to [8] for the detailed derivations of the covariance matrices for $\underline{N}_1(i)$ and $\underline{I}_1(i)$.

Since mobile 1 has no knowledge of the other users' transmit weight vectors $\{\underline{v}_k\}$, an ad hoc criterion of generating its own transmit weight vector $\underline{v}_1$ is to maximize the effective received signal power

$$P_r^{(1)} = \left|\underline{w}_1^H G_1\underline{v}_1\right|^2 \le \left(\left|(G_1)^H\underline{w}_1\right|^2\right)\left(|\underline{v}_1|^2\right) \tag{12}$$

where equality holds if and only if $\widehat{\underline{v}}_1 = c_1(G_1)^H\underline{w}_1$, and c_1 is a constant for normalization. If the transmitter has perfect knowledge of channel state information G_1 and the corresponding receive weight vector $\underline{w}_1$, $P_r^{(1)}$ can be maximized by setting $\widehat{\underline{v}}_1 = c_1(G_1)^H\underline{w}_1$. This concept for determining the transmit weight vector is known as maximal ratio transmission. Subject to the power constraint $\|\underline{v}_1\|^2 = 1$, the transmit weight vector is given by

$$\underline{v}_1 = \frac{(G_1)^H\underline{w}_1}{\|(G_1)^H\underline{w}_1\|}. \tag{13}$$

Now the goal is to choose the beamforming weight vector $\underline{w}_1$ which maximizes the received power for the desired user. Subject to the normalization constraint, the optimum receive weight vector $\underline{w}_1$ is obtained as

$$\widehat{\underline{w}}_1 = \arg \max_{\|\underline{w}_1\|^2=1} \left\{ \left| (\underline{w}_1)^H G_1 (G_1)^H \underline{w}_1 \right|^2 \right\}. \quad (14)$$

Therefore, the receive weight vector $\widehat{\underline{w}}_1$ is the principal eigenvector of $G_1(G_1)^H$, and the received power is the corresponding eigenvalue, i.e., the maximum eigenvalue $\widehat{\lambda}$ of $G_1(G_1)^H$.

NUMBERICAL RESULTS AND DISCUSSIONS

We assume that the fading seen by each transmit antenna is independent, since L_T is usually a small number. At the receiver, L_R receive antennas are deployed for adaptive beamforming reception, where L_R can be a large number so that the fading experienced by each receive antenna might be correlated. L_T independent transmit antennas and L_R independent receive antennas give $L_T \cdot L_R$ order spatial diversity gain. So fixing the value of $L_T \cdot L_R$ fixes the maximum diversity order achievable by the system. When the fading is, in fact, correlated, the diversity gain from the receive antenna array is reduced. However, independent fading is not required for interference suppression, since correlated receive antennas can still be used for MAI suppression. Suppose we fix the product $L_T \cdot L_R$, just for the sake of having a frame of reference for a tradeoff. Then increasing L_T will increase the diversity gain against fading while sacrificing some of the receive antenna array's capability of MAI suppression. This will be illustrated below.

We assume the use of a raised-cosine filter characteristic, with rolloff factor $\alpha = 0.5$, for pulse shaping. We further assume the processing gain to be fixed at $N_s = 64$. Since it is difficult to analytically derive the pdf of the instantaneous SNR, $f_\gamma(\gamma)$, we cannot obtain a closed-form expression for the BER. To circumvent this problem, a Monte-Carlo simulation is carried out. After one million trials, the SNR distribution of the combined outputs at the receiver is accumulated and $f_\gamma(\gamma)$ is numerically determined. The SNR value γ for each combined output is applied to the conditional bit error probability of a BPSK system, $\phi(\sqrt{2\gamma})$, and the average BER is calculated by integrating $Pe = \int_0^\infty \phi(\sqrt{2\gamma}) f_\gamma(\gamma) d\gamma$.

In Fig. 4, we consider a DS-CDMA system with 10 users, where the interference power is log-normally distributed with a $3dB$ standard deviation. The average BER versus E_b/η_0 with different sets of parameters is shown in the figure. With $L_T \cdot L_R$ fixed to be 8, we find that the system employing 2 transmit antennas and 4 receive antennas is better than one employing 1 transmit antenna and 8 receive antennas. At a BER of 10^{-3}, there is nearly a $5dB$ enhancement by setting $L_T = 2$ and $L_R = 4$. This is primarily due to the two-fold diversity gain from the two transmit antennas with independent fading. Note that since the total length of the receive array is fixed at a value such that the multiple receive antennas experience correlated fading, the resulting effective diversity order achieved by the 8-antenna array is less than twice that achieved by the 4-antenna array, although the MAI suppression capability is enhanced with more receive antennas.

We further evaluate the system performance with a more severe near-far problem, i.e, interference power is log-normally distributed with a $10dB$ standard deviation. Compared to the system with better power control, the BER performance of both of the above systems degrades. It is observed that the degradation is more significant for the $L_T = 2$ and $L_R = 4$ system than it is for the $L_T = 1$ and $L_R = 8$ system. The system's ability to suppress MAI is augmented by using more receive antennas, while sacrificing some diversity gain from the transmit antennas, and in the presence of a large amount of MAI, this interference suppression is more important than the diversity gain against fading.

In Fig. 5, we plot the BER performance curves for systems with $Ku = 20$. Compared with the curves plotted in Fig. 4 for systems with $Ku = 10$, there is smaller degradation when $L_R = 8$ receive antennas are employed. However, the degradation is much more conspicuous when only $L_R = 4$ receive antennas are used. Since MAI becomes more dominant compared to the fading when the number of interferers increases, and the system with 8 antennas is more capable of MAI suppression, the performance gap between the two systems decreases. Furthermore, for the system with 20 users and/or $10dB$ standard deviation, the performance curves of $L_T = 2$, $L_R = 4$ and $L_T = 1$, $L_R = 8$ are quite close.

CONCLUSION

In this paper, we proposed a DS-CDMA system employing multiple antennas at both the mobile and the base station. Maximal ratio transmission and adaptive beamforming reception are used to achieve the maximum received power for the desired user in a multiple access channel with correlated Rayleigh fading. The conditional SNR is analytically derived and the average BER is investigated via simulation. We examined the effect of varying the number of transmit and receive antennas on both the diversity gain against fading and the MAI suppression. The benefit of using only a single transmit antenna is easier implementation in a small mobile unit. However, when the number of active users is stable and/or accurate power control is maintained, using a larger number of independent transmit antennas is preferred.

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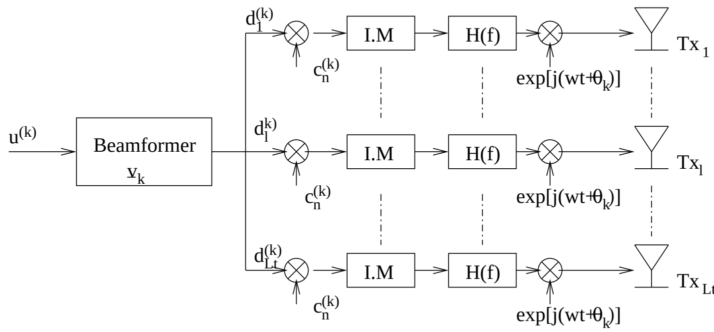

Fig. 1. Transmitter for MRT in a CDMA system

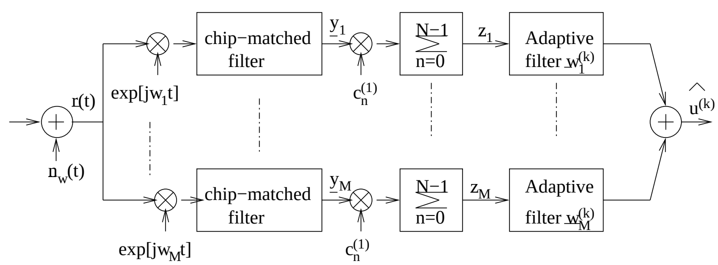

Fig. 2. Receiver with adaptive beamforming in a CDMA system

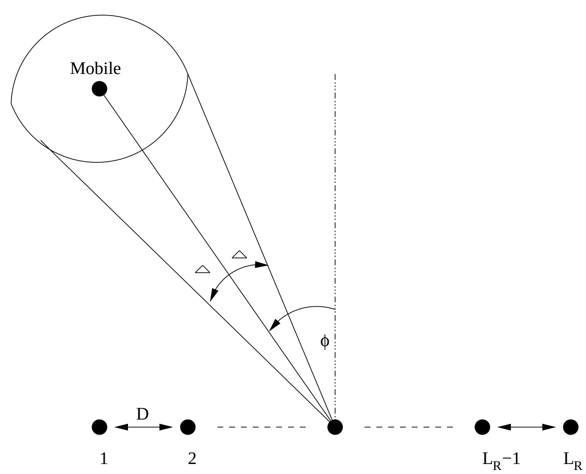

Fig. 3. Wireless environment where all signals from a transmit antenna arrive at the BS within $\pm\Delta_l$ of angle Φ_l

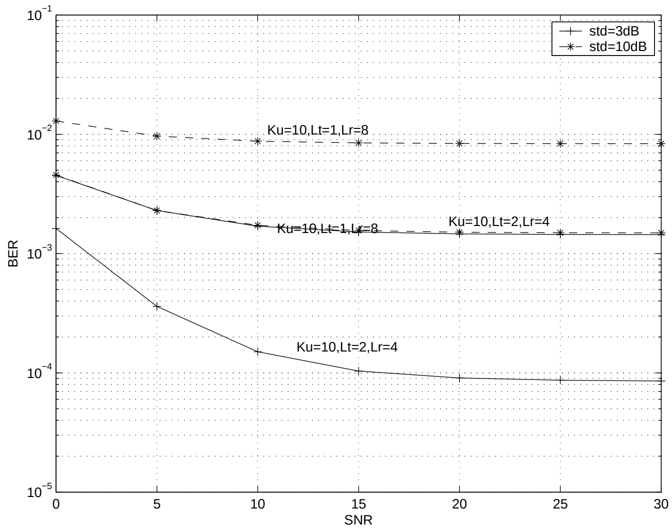

Fig. 4. BER versus E_b/η_0 with $K_u = 10$ and $L_T \cdot L_R = 8$

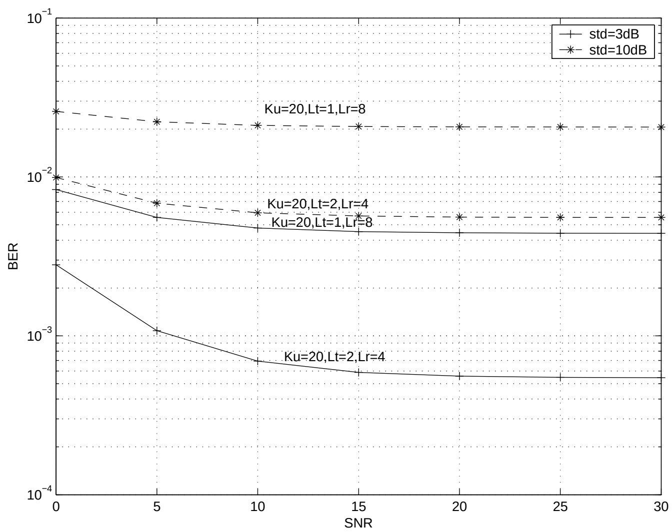

Fig. 5. BER versus E_b/η_0 with $K_u = 20$ and $L_T \cdot L_R = 8$