

# Serial Concatenated Trellis Coded Modulation with Inner Rate-1 Accumulate Code

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**Abstract**— In this paper we propose a Serial Concatenated Trellis Coded Modulation system using one or more inner accumulate code(s) and a Gray-labeled signal constellation. We show the existence of a threshold, such that if the signal-to-noise ratio (SNR) exceeds this threshold, the bit error probability goes to zero as the blocklength goes to infinity. Tight numerical values for the thresholds for an iterative decoder are found by density evolution. Despite the simple inner code, the simulated performance in AWGN and Rayleigh fading is comparable to that of more complex systems suggested in the literature.

## I. INTRODUCTION

Several Turbo-TCM systems have been proposed to merge the extraordinary performance of turbo codes [1] with the bandwidth efficiency of Trellis-Coded Modulation (TCM) [2]. Both Parallel Concatenated TCM (PCTCM) [3], [4] and Serial Concatenated TCM (SCTCM) [5], [6] have been shown to achieve good performance. Bit-Interleaved Coded Modulation (BICM), initially proposed to increase the diversity over fading channels [7], with iterative decoding (BICM-ID) gives almost the same performance as Turbo-TCM over AWGN channels, but at a lower complexity [8].

Inspired by the analytical tractability of Repeat-Accumulate (RA) codes [9] and their generalizations [10], we propose an SCTCM system where the inner code is one or more accumulate codes, followed by a mapping to a higher-order, Gray-labeled constellation. The accumulate code is a rate-1 code with the rational transfer function  $G(D) = 1/(1 \oplus D)$ . For higher spectral efficiency, we use Parity-Accumulate (PA) codes, where the outer code is a Single Parity Check (SPC) code, or a system with an outer high-rate convolutional code.

For Maximum-Likelihood (ML) decoding of the proposed SCTCM system, there exists a *threshold*, a minimum SNR  $\gamma_{ML}^*$ , such that for all SNR  $\gamma > \gamma_{ML}^*$ , the bit error probability  $P_b \rightarrow 0$  as the blocklength  $N \rightarrow \infty$ . We show the existence of such threshold by extending the union bound technique of [9], [10] to higher-order constellations. The union bound technique gives large numerical values for the threshold  $\gamma_{ML}^*$ . By applying Density Evolution (DE), we find numerical values for the threshold  $\gamma_{It}^*$  for an iterative, non-ML decoder. For PA codes,  $P_b$  decreases very slowly as the number of decoding iterations increases. It is therefore hard to determine a numerical value for the threshold for PA codes via DE. Instead, we derive a stability criterion similar to the one in [11] to find a lower bound on the numerical value for the threshold for PA codes.

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We conclude the paper by comparing the numerical values for the thresholds to simulations. We also compare the performance of the proposed SCTCM system, with 8-PSK and 16-QAM modulation over AWGN and Rayleigh fading channels, to SCTCM and BICM-ID systems [6], [8].

## II. SYSTEM DESCRIPTION

### A. Encoder

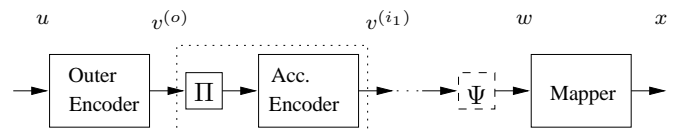


Fig. 1. The basic encoder uses a single inner accumulate code and no interleaver  $\Psi$  before the mapper. The generalized system uses multiple accumulate codes and non-trivial interleaver  $\Psi$ .

The encoder is shown in Fig. 1. The outer encoder is a block code, formed either by terminating a rate  $r_c = k/n$  convolutional code or by concatenating short block codes, such as  $r_c = (n - 1)/n$  SPC codes or  $r_c = 1/n$  repeat codes. The interleaver  $\Pi$  is a random or S-random interleaver and acts on all bits in a block. The size of the interleaver is  $N$  and it is assumed that  $n$  divides  $N$ . The inner code is a rate-1 accumulate code. The memoryless mapper maps an  $m$ -tuple of bits to a constellation point  $x \in \mathcal{X}$ , where  $\mathcal{X}$  is a Gray-labeled constellation of size  $|\mathcal{X}| = M = 2^m$ .

The generalization of the encoder to include multiple interleaved accumulate codes is straightforward. The interleaver and accumulate code, which comprise the dotted section in Fig. 1, are repeated the desired number of times. Furthermore, we can introduce a non-trivial interleaver,  $\Psi$ , before the mapper. With an outer convolutional code, no accumulate codes, and a non-trivial interleaver  $\Psi$ , we get the BICM system of [7].

### B. Decoder

The decoder, depicted in Fig. 2, consists of a bit metric calculator, soft-input soft-output (SISO) module(s) matched to the inner code(s) and an SISO module matched to the outer code, separated by appropriate interleavers and deinterleavers. The final, binary decision is given by a slicer.

The output of the metric computation is the extrinsic probability that the  $i$ -th bit of the  $m$ -tuple  $w$  has the value  $b$ , given all available information except the information about the  $i$ -th bit itself. If the interleaver  $\Psi$  is non-trivial, the bits that make

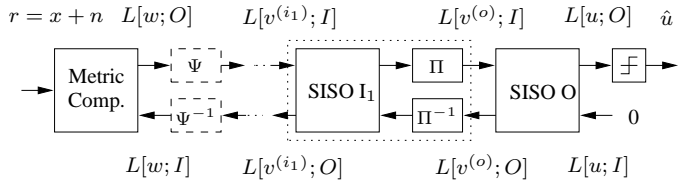


Fig. 2. The basic decoder has two SISO modules and does not feed back any a priori information to the metric computation. The general decoder can include more SISO modules and may feed back a priori information.

up the symbol  $x$  are independent. The extrinsic probability is then

$$P[w^i = b; O] = \sum_{x \in \mathcal{X}_b^i} P[r|x] \prod_{\substack{j=1 \\ j \neq i}}^m P[w^j = \ell^j(x); I], \quad (1)$$

where  $\mathcal{X}_b^i = \{x \in \mathcal{X} | \ell^i(x) = b\}$  is the set of constellation points in the constellation  $\mathcal{X}$  such that the  $i$ -th bit,  $i = \{1, \dots, m\}$ , in the binary label of the point  $x$  has the value  $b$ ,  $b = \{0, 1\}$ . Log-likelihood ratios (LLR) are formed,  $L[w; O] = \log(P[w = b; O]) - \log(P[w = \bar{b}; O])$ , and fed to the innermost SISO.

### III. CODING THEOREMS

In this section we show the existence of an SNR threshold  $\gamma^*$  for ML decoding of the system in Fig. 1. We consider the reciprocal of the SNR  $\gamma$ , the channel parameter  $z$ , and find the channel parameter threshold  $z^*$ . By extending the union bound technique of [9], [10] to higher-order constellations, we give coding theorems stating that if  $z < z^*$ , then, for the system using RA codes, the word error probability  $P_W$  goes to zero as  $N \rightarrow \infty$ , and, for the system using PA codes, the bit error probability  $P_b$  goes to zero as  $N \rightarrow \infty$ .

The union bound on  $P_W$  is a function of the weight enumerator (WE) for the concatenated codes,  $A_h^{(c)}$ , the output weight  $h$ , and the channel parameter  $z$ ,

$$P_W \leq \sum_{h>0} A_h^{(c)} z^h \leq \sum_{h>0} e^{h(F(\cdot) + \ln z)}. \quad (2)$$

By expressing the bound on  $P_W$  in the latter form, where  $F(\cdot)$  is a function of the WE, we conclude that if  $\ln z < -F(\cdot)$ , then  $P_W \rightarrow 0$  as  $N \rightarrow \infty$ .

The channel parameter  $z$  for a 2-dimensional, Gray-labeled constellation of size  $2^m$  over an AWGN channel is

$$z = e^{-\frac{r_c m E_b}{4 N_0} d_{\min}^2}, \quad (3)$$

where  $d_{\min}^2$  is the minimum squared Euclidean distance (SED) for the constellation and  $E_b/N_0$  is the bit signal-to-noise ratio.

For an independently fading Rayleigh channel, we get

$$z = \left[ \frac{1}{1 + \frac{r_c m d_{\min}^2 E_b}{4 N_0}} \right]^L, \quad (4)$$

where  $L$  is the normalized diversity order,  $1/m \leq L \leq 1$ . The performance is dominated by codewords with small weight  $h$ , and it can be assumed that  $L = 1$ .

For serial concatenation of two codes through a uniform interleaver, the Input Output Weight Enumerator (IOWE) is given by

$$A_{w,h}^{(c)} = \sum_{h_o=0}^N \frac{A_{w,h_o}^{(o)} A_{h_o,h}^{(i)}}{\binom{N}{h_o}}, \quad (5)$$

where  $A_{w,h_o}^{(o)}$ ,  $A_{h_o,h}^{(i)}$  are the IOWE coefficients for the outer and inner codes, and  $w$  and  $h_o$  are the input and output weights for the outer code. The IOWEs for repeat codes and accumulate codes are given in [9]. For a rate  $r_c = (n-1)/n$  even SPC code, we derive an upper bound on the WE,

$$A_h \leq \binom{\lfloor n/2 \rfloor \cdot N/n}{h/2} \binom{n}{\lfloor n/2 \rfloor}^{h/2}. \quad (6)$$

The second factor in (6) can be approximated by  $\binom{n}{\delta n}$ , where  $\delta \triangleq h/N$ .

#### A. Repeat-Accumulate Codes

For RA codes, we show the following result:

*For an outer repeat code of rate  $r_c \leq 1/3$ , followed by a single accumulate code and a Gray-labeled constellation, there exists a  $z^*$ , such that for  $z < z^*$ ,  $P_W \rightarrow 0$  as  $N \rightarrow \infty$ .*

This follows from a straightforward application of the technique in [9], combined with the channel parameters in (3) and (4). The corresponding  $E_b/N_0$ -thresholds,  $\gamma^*$ , above which  $P_W \rightarrow 0$  as  $N \rightarrow \infty$  are given in Table I.

TABLE I

$E_b/N_0$ -THRESHOLDS IN dB FOR  $P_W$  FOR RA CODES.

rate	AWGN		Fading	
	8-PSK	16-QAM	8-PSK	16-QAM
1/3	5.77	6.18	7.03	7.44
1/4	5.50	5.91	6.37	6.78

#### B. Parity-Accumulate Codes

For PA codes, we show that:

*For an outer SPC code, and a single inner accumulate code, there exists a  $z^*$  such that for  $z < z^*$ ,  $P_b \rightarrow 0$  as  $N \rightarrow \infty$ .*

Since the free distance for the outer SPC code  $d_{\text{free}}^{(o)} = 2$ ,  $P_W$  does not go to zero as  $N$  goes to infinity [12]. However, by expressing the upper bound on  $P_b$  as

$$P_b \leq \sum_{h=1}^N \sum_{h_o=0}^{2h} \frac{h_o}{rN} \frac{A_{h_o}^{(o)} A_{h_o,h}^{(i)}}{\binom{N}{h_o}} z^h \quad (7)$$

we can show that  $P_b \rightarrow 0$  as  $N \rightarrow \infty$ . In the proof, we use several loose upper bounds and the resulting numerical values

are quite pessimistic. If we, instead of using upper bounds, approximate the second factor of (6) and approximate the binomial coefficients in the other WEs, we get numerical values that are not true upper bounds on the threshold  $\gamma^*$ , but can give an indication of what to expect from simulations. The  $E_b/N_0$ -thresholds computed with the upper bound (ub) and the approximations are given in Table II for some different code rates over AWGN and fading channels.

Despite approximations, comparison of the thresholds in Tables I and II to simulations shows that the thresholds are loose.

TABLE II  
 $E_b/N_0$ -THRESHOLDS IN dB FOR  $P_b$  FOR PA CODES.

rate	8-PSK		16-QAM			
	AWGN		AWGN		Fading	
	ub	appr.	ub	appr.	appr.	
1/2	18.28	9.59	14.64	18.69	10.00	15.04
2/3	20.04	11.35	22.62	20.45	11.76	23.03
3/4	28.56	15.06	50.72	28.97	15.47	51.13
8/9	42.87	17.85	107.36	43.28	18.25	107.76

#### IV. DENSITY EVOLUTION

In this section, we apply density evolution (DE) to the proposed SCTCM system to find tighter numerical values for the threshold  $\gamma^*$ . The idea of DE is to track how the densities of the information bits (or their LLR) evolve as the number of decoding iterations increases [13]. In the following “density” or “pdf” refers to the pdf for the LLR.

##### A. Channel Densities

For BPSK modulation over AWGN channels, the LLR is a linear function of the received value. It is straightforward to show that the LLR pdf is a Gaussian with mean  $E[L] = \frac{2x^2}{\sigma^2}$  and variance  $\text{var}[L] = \frac{4x^2}{\sigma^2} = 2E[L]$ . For a Rayleigh fading channel, the conditional pdf is Gaussian and the unconditional pdf is given by

$$p_L(l|x) = \frac{e^{\frac{1}{2}\left(l - \sqrt{2 + \frac{x^2}{\sigma^2}} \frac{\sigma}{x} \sqrt{l^2}\right)} \sigma}{2x \sqrt{2 + \frac{x^2}{\sigma^2}}}. \quad (8)$$

For higher order constellations, the pdfs for the  $m$  bits can be different and may also depend on which constellation point was transmitted. The LLR for the  $i$ -th bit,  $i \in \{1, \dots, m\}$ , when the  $j$ -th symbol is transmitted,  $j \in \{1, \dots, M\}$ , is

$$l_{i,j} = \log \frac{\sum_{x \in \mathcal{X}_{\ell^i(x_j)}^i} p(r|x_j)}{\sum_{x \in \mathcal{X}_{\ell^i(x_j)}^i} p(r|x_j)}, \quad (9)$$

where  $r$  is the received value and  $\mathcal{X}_{\ell^i(x_j)}^i = \{x \in \mathcal{X} | \ell^i(x) = \ell^i(x_j)\}$  is the set of constellation points where the  $i$ -th bit of the binary label has the same value as the  $i$ -th bit in the label of

the transmitted point,  $x_j$ . We find the corresponding pdf  $p_{L_i}(l)$  by numerical evaluation. Assuming all points are transmitted equiprobably, we get the pdf for bit  $i$  as

$$p_{L_i}(l) = \frac{1}{M} \sum_{j=1}^M p_{L_{i,j}}(l). \quad (10)$$

##### B. Code Graph

The graph for a PA code is shown in Fig. 3. At the bit nodes, drawn as circles, random variables (r.v.s) are added, and hence the pdfs are convolved (denoted  $\otimes$ ). At the parity check nodes, drawn as squares, r.v.s are added under a parity check constraint, which leads to the well-known tanh-atanh operation

$$x \boxplus y = 2 \operatorname{atanh}(\tanh(x/2) \cdot \tanh(y/2)). \quad (11)$$

Since the inner code is a 2-state accumulate code, the graph has hidden nodes, drawn as double circles. The hidden nodes hold the forward and backward state messages, denoted by  $\alpha$  and  $\beta$ , respectively.

If a non-trivial interleaver  $\Psi$  is present between the accumulate code and the mapper (bottom interleaver in Fig. 3), we average the  $m$  pdfs from the channel before the DE

$$p_L(l) = \frac{1}{m} \sum_{i=0}^{m-1} p_{L_i}(l). \quad (12)$$

In this case all  $\alpha$  in the accumulate code are the same, as are the  $\beta$  and  $u$  messages. The update equations for the DE are

$$\alpha_1 \leftarrow (\alpha_1 \boxplus d_1) \otimes p_L \quad (13)$$

$$\beta_1 \leftarrow (\beta_1 \otimes p_L) \boxplus d_1 \quad (14)$$

$$u_1 = (\beta_1 \otimes p_L) \boxplus \alpha_1 \quad (15)$$

$$d_1 = u_1 \boxplus u_1 \dots \boxplus u_1. \quad (16)$$

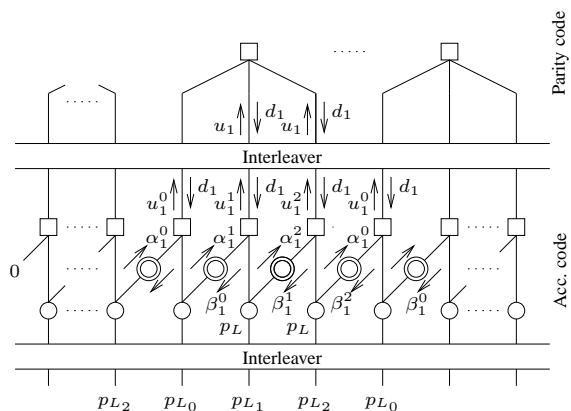


Fig. 3. Decoding graph for a rate-2/3 PA code, shown with two interleavers.

If there is no interleaver between the accumulate code and the mapper, then there are  $m$  different  $\alpha$ ,  $\beta$  and  $u$  messages. Due to the interleaver between the accumulate code and SPC,

the upward message,  $u$ , is averaged before being passed to the SPC code.

For multiple accumulate codes, the accumulate code layer is repeated.

## V. STABILITY CONDITION

When  $\gamma > \gamma^*$ ,  $P_b$  decreases rapidly as a function of the decoding iterations for RA and  $PA^i$ ,  $i \geq 2$ , codes (where the superscript denotes the number of inner accumulate codes). For  $PA^1$  codes, this is not the case and it is hard to determine a numerical value for the threshold. In this case, we use a technique similar to the one in [11] to find a lower bound on the threshold. We use a simplified graph, in that we do not consider the state information, see Fig. 4.

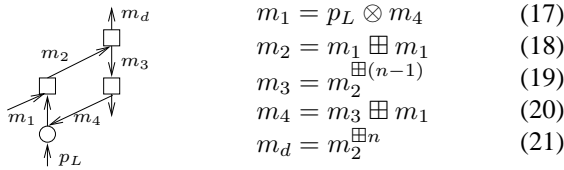


Fig. 4. Fragment of the simplified decoding graph for a PA code.

The messages are mixed pdfs with one discrete part,  $\delta_\infty$ , at  $+\infty$ , and one continuous part,  $Q(l)$ ,  $\int_{-\infty}^{\infty} Q(l) dl = 1$ ,

$$m(l) = (1 - \varepsilon)\delta_\infty + \varepsilon Q(l), \quad (22)$$

where  $1 - \varepsilon$  is the fraction of the pdf at  $+\infty$  and  $\varepsilon$  is the fraction in the continuous part. Solving for the decision message,  $m_d$ , after  $i$  iterations gives

$$m_d \approx (1 - 2n(2n - 1)^i \varepsilon)\delta_\infty + 2n(2n - 1)^i \varepsilon p_L^{\otimes i} \otimes Q. \quad (23)$$

The term  $\delta_\infty$  does not contribute to the error probability, so  $P_b$  is given by

$$P_b = \int_{-\infty}^0 2n(2n - 1)^i \varepsilon p_L(l)^{\otimes i} \otimes Q(l) dl. \quad (24)$$

We can not evaluate (24), since it contains two unknowns,  $\varepsilon$  and  $Q(l)$ . However, if

$$\int_{-\infty}^0 (2n - 1)^i p_L(l)^{\otimes i} dl \quad (25)$$

does not go to zero as  $i \rightarrow \infty$ , then  $P_b$  is bounded away from 0. Finding  $p_L(l)$  such that the integral in (25) goes to zero as  $i \rightarrow \infty$  gives us a lower bound on the threshold  $\gamma^*$ .

## VI. NUMERICAL RESULTS

We report some numerical results and compare to the channel capacity. For constrained input channels, such as PSK and QAM, and equiprobable signaling, the capacity  $C^*$  for AWGN is computed as in [2, eq. (5)]. The extension to independent Rayleigh fading channels is straightforward.

The DE thresholds are computed assuming infinite blocklength, i.e., all messages are independent.

In Tables III and IV we list the decoding thresholds for RA and PA codes with 8-PSK and 16-QAM modulation for AWGN and independent Rayleigh fading channels. Rates  $r_c < 1/2$  are outer repeat codes and rates  $r_c > 1/2$  are outer SPC codes. For  $r_c = 1/2$ , repeat and SPC codes are the same. The threshold with  $i$  accumulate codes is denoted  $\gamma_i^*$ . The thresholds are computed with the message density averaged after the first layer of accumulate codes. The thresholds for  $PA^i$ ,  $i = 2, 3$  and RA codes are computed by density evolution and the thresholds for  $PA^1$  codes are computed with the stability method described in Section V.

TABLE III

$E_b/N_0$  THRESHOLDS IN dB ABOVE WHICH  $P_b \rightarrow 0$  AS  $N \rightarrow \infty$  FOR 8-PSK MODULATION OVER AWGN AND FADING CHANNELS.

rate	AWGN				Fading			
	C*	$\gamma_1^*$	$\gamma_2^*$	$\gamma_3^*$	C*	$\gamma_1^*$	$\gamma_2^*$	$\gamma_3^*$
1/4	-0.36	0.97	3.49	6.34	0.62	2.04	5.18	8.89
1/3	0.12	1.44	3.01	5.56	1.41	2.83	4.89	8.25
1/2	1.28	5.29	2.78	4.69	3.18	7.89	5.05	7.68
2/3	2.75	6.18	3.83	5.18	5.37	9.82	6.88	9.00
3/4	3.66	6.71	4.54	5.63	6.81	11.12	8.17	10.09
8/9	5.66	7.93	6.21	6.87	10.61	14.66	11.73	13.34

TABLE IV

$E_b/N_0$  THRESHOLDS IN dB ABOVE WHICH  $P_b \rightarrow 0$  AS  $N \rightarrow \infty$  FOR 16-QAM MODULATION OVER AWGN AND FADING CHANNELS.

rate	AWGN				Fading			
	C*	$\gamma_1^*$	$\gamma_2^*$	$\gamma_3^*$	C*	$\gamma_1^*$	$\gamma_2^*$	$\gamma_3^*$
1/4	0.07	1.61	4.13	7.13	1.16	2.63	5.97	9.68
1/3	0.69	2.22	3.72	6.37	2.07	3.48	5.66	9.04
1/2	2.11	6.15	3.60	5.51	3.93	8.64	5.80	8.40
2/3	3.68	6.96	4.67	5.97	6.13	10.55	7.62	9.72
3/4	4.54	7.43	5.34	6.36	7.57	11.84	8.90	10.80
8/9	6.36	8.54	6.88	7.51	11.33	15.37	12.44	14.04

## VII. SIMULATION RESULTS

In Fig. 5 we compare simulated performance of RA and PA codes to the thresholds of Table III for 8-PSK modulation over AWGN. The blocklength for the RA code is  $N = 10^5$  and for the PA code  $N = 7.8 \times 10^5$ . Performance after 1, 10, 20 and 50 iterations is shown.

In the following, we use an outer convolutional code, a single accumulate code and no interleaver  $\Psi$  before the mapper.

In Fig. 6 we show the performance in AWGN and compare to the SCTCM scheme in [6]. Both systems use an outer  $r_c = 3/4$ , memory-2 convolutional code and 16-QAM modulation, and the spectral efficiency is 3 bits/s/Hz. The blocklength is 12288 information symbols, corresponding to 4096 channel symbols. The system in [6] also uses an inner rate-1 code, albeit more complex and jointly optimized with the constellation labeling. Our proposed system uses a simpler inner

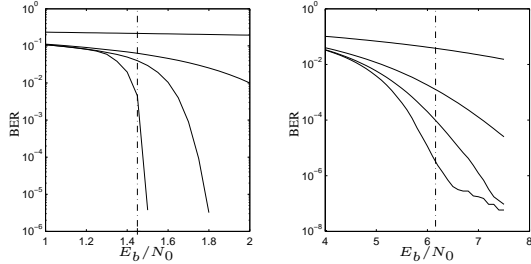


Fig. 5. Performance of  $r_c = 1/3$  RA code (left) and  $r_c = 2/3$  PA code (right) over AWGN channel using 8-PSK modulation. The dot-dashed lines are the DE thresholds.

code, but still achieves the same performance.

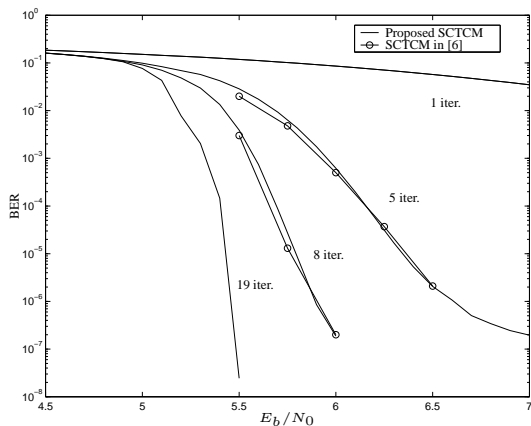


Fig. 6. Performance comparison in AWGN. 16-QAM constellation and block-length 4096 channel symbols. After 20 iterations we have not seen errors for  $E_b/N_0 > 5.4$  dB.

As BICM was initially proposed for a fading channel, we compare the performance of the proposed SCTCM system to BICM-ID [8] over a flat Rayleigh fading channel, see Fig. 7. Simulation results are shown for an input block size 32768 bits, outer  $r = 2/3$  convolutional code,  $d_{\text{free}}^o = 4$  and interleaver size  $N = 49152$ . At high bit error rates (BER), BICM-ID performs slightly better, but at low BER the proposed SCTCM system is advantageous. Furthermore, BICM-ID exhibits an error floor, in fading as well as in AWGN [8].

## VIII. CONCLUSIONS

An SCTCM system with inner accumulate code(s) has been introduced. We have shown that the simplicity of the system allows us to state coding theorems for infinite blocklengths. We have devised a method to compute the LLR pdfs for higher order constellations. We have computed thresholds for an iterative decoder using density evolution and a stability criterion. For PA<sup>2</sup> codes over AWGN, the thresholds are about a dB away from the constrained capacity for  $r_c = 2/3$ , and closer to capacity for higher rates.

Though the proposed system uses a simple inner accumulate

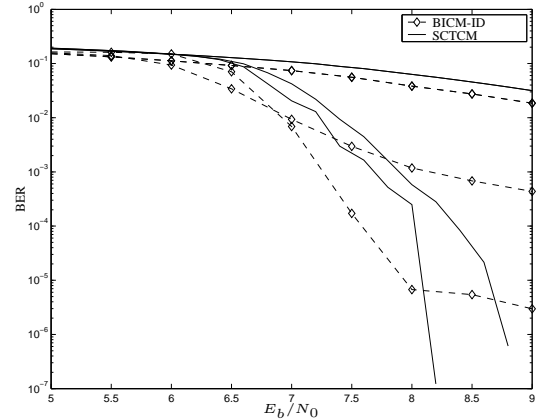


Fig. 7. Performance comparison in fading. Performance after 1, 5 and 16 iterations shown.

code, in AWGN it performs comparably to serial concatenation schemes recently reported in the literature [6] that use more complex rate-1 inner codes. Over a fading channel, the proposed SCTCM system performs better than BICM-ID at low bit error rates, since the proposed system has a much steeper error floor.

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